

Regla de Cramer

Sistemas 2×2 :

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\underbrace{A}_{n \times n} x = b \rightarrow \text{No singular} \\ |A| \neq 0$$

$$\Delta = |A| = \det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = b_1 a_{22} - b_2 a_{12}$$

$$\Delta_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = a_{11} b_2 - a_{21} b_1$$

$$x_1 = \frac{\Delta_1}{\Delta}$$

$$x_2 = \frac{\Delta_2}{\Delta}$$

Sistemas 3x3:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\Delta = \begin{matrix} + & - & + \\ \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \end{matrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\Delta = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

$$\Delta_1 = \begin{vmatrix} + & - & + \\ b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta_1 = b_1 (a_{22} a_{33} - a_{32} a_{23}) - a_{12} (b_2 a_{33} - b_3 a_{23}) + a_{13} (b_2 a_{32} - b_3 a_{22})$$

$$\Delta_2 = \begin{vmatrix} + & - & + \\ a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

$$\Delta_2 = a_{11} (b_2 a_{33} - b_3 a_{23}) - b_1 (a_{21} a_{33} - a_{31} a_{23}) + a_{13} (a_{21} b_3 - a_{31} b_2)$$

$$\Delta_3 = \begin{array}{c} + \quad - \quad + \\ \left| \begin{array}{ccc} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{array} \right| \end{array}$$

$$\Delta_3 = a_{11}(a_{22}b_3 - a_{32}b_2) - a_{12}(a_{21}b_3 - a_{31}b_2) + b_1(a_{21}a_{32} - a_{31}a_{22})$$

$$x_1 = \frac{\Delta_1}{\Delta} \quad x_2 = \frac{\Delta_2}{\Delta} \quad x_3 = \frac{\Delta_3}{\Delta}$$